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DISCUSSION OF
STRESSES IN DEEP BEAMS

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By Arturo M. Guzmán and Cesar J. Luisoni;
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Conway and George Winter

STRUCTURAL DIVISION

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DISCUSSION

ARTURO M. GUZMÁN,¹⁶ AND CESAR J. LUISONI.¹⁷—Special interest has been aroused by the subject because of the importance of the deep beam as a strong structural element in reinforced concrete industrial buildings and in large residential blocks. In France, numerous detailed prescriptions have been issued by the government on this matter.¹⁸

The analysis of a single-span deep beam is particularly difficult. For their calculations, the authors use the method of finite differences, and the agreement of the approximation depends, of course, on the density of the network that has been adopted. For the case $H = L$ with supporting width $0.1 L$ and the load on the top edge, there exist complete photoelastic tests using the very exact, purely optical, Favre method.¹⁹ In the central section, for the maximum bending stress, σ_x , the difference is 20% at the bottom edge; and the finite differences method, with the network width as indicated by the authors, yields smaller values than the experimental method.

A more accurate approximation, involving less calculation,^{20,21} is possible by using fifth-degree polynomials for the Airy stress function and, therefore, third-degree polynomials for σ_x , the coefficients for the polynomials being determined by the Galerkin variational method. This solution has then been extended to several cases of load,^{22,23} with tabular data being given for each. In the case analyzed by the authors, using finite differences, fifteen equations with as many unknowns must be solved, whereas with the Galerkin variational method used by the writers, only four equations with as many unknowns are required, the agreement with the photoelastic test being very good.

For the case $H = 1.5 L$, uniformly loaded, the writers have used the finite differences method with a very dense network, $h = k = L/8$ or 44 points, solving the equations by the Southwell relaxation method^{22,23} and using the so-called block relaxation to speed up the convergence; even so, however, the calculation work is considerable. Nevertheless, differences of about 10% to 20% as compared with experimental values²⁴ always arise. The rectangular network used by H. Bay⁹ and the authors is not to be recommended as it

NOTE—This paper by Li Chow, Harry D. Conway, and George Winter, was published in May, 1952, as *Proceedings-Separate No. 127*. The numbering of footnotes and equations in this Separate is a continuation of the consecutive numbering used in the original paper.

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¹⁸ "Traité de Béton Armé," by A. Guerrin, *Règles B. A. 45 du Ministère de la Reconstruction et de l'Urbanisme (1945-1948)*, Paris, France, Vol. II, 1951, pp. 208-216.

¹⁹ "Ensayo fotoelástico de una viga de gran altura," by C. A. Sciammarella and M. A. Palacio, *Ciencia y Técnica*, Buenos Aires, Argentina, Vol. 113, No. 569, 1949.

²⁰ "Solución variacional del problema de la viga rectangular simplemente apoyada de gran altura," by A. M. Guzmán and C. J. Luisoni, *Ciencia y Técnica*, Buenos Aires, Argentina, Vol. 111, No. 555, 1948.

²¹ *Applied Mechanics Reviews*, ASME, Vol. 1, November, 1948, Revs. 1608.

²² "Sobre la viga simplemente apoyada de gran altura. Teoría y experimentación," by A. M. Guzmán and C. J. Luisoni, *Ciencia y Técnica*, Buenos Aires, Argentina, Vol. 114, No. 576, 1950.

²³ *Applied Mechanics Reviews*, ASME, Vol. 3, October, 1950, Revs. 1807.

²⁴ "Ensayo fotoelástico de una viga de gran altura," by C. A. Sciammarella and M. A. Palacio, *Ciencia y Técnica*, Buenos Aires, Argentina, Vol. 117, No. 588, 1951.

⁹ "Über den Spannungszustand in hohen Trägern und die Bewehrung von Eisenbetontragwänden," by H. Bay, Stuttgart, Germany, 1931, p. 64.

increases the error. This defect is discernible in Fig. 4 for $H = 2L$, where the surface of the compression zone is about 70% superior to the tension zone for σ_x , whereas both must be equal for calculation control and static conditions.

For uniform load, the maximum bending stress at the bottom edge does not occur in the middle but on the sides. This fact cannot appear in finite differences, but is revealed in the variational method, and has been confirmed by photoelastic tests.

The behavior of tensile reinforcement, for single-span or two-span reinforced concrete deep beams, has not been studied extensively. Special mention should be made of a contribution by H. Nylander and H. Holst²⁵ that is not sufficiently known, and a recent discussion in which experiments by Garcia Olano-Fliess²⁶ are cited. The Swedish authors²⁵ give interesting conclusions regarding the cracking mechanism of deep beams with a load on the top edge, which can be forecast by studying theoretically derived isostatics or by photoelasticity. Messrs. Nylander and Holst²⁷ state:

"* * * after the formation of tensile cracks the member acts as a straining trestle-work provided with tension rods, and the load-bearing capacity is therefore dependent on the anchorage of the bending tensile reinforcement. For this reason, the shear reinforcement should be proportioned so as to provide safety against the formation of detrimental cracks."

Tests²⁶ for a single-span deep beam with a load at the top edge, and for $H = 1.5L$, reveal that the failure occurs when the member has lost its resistance to compression in the supports and that the tension in the reinforcement at failure is very low. The cracks always start on the interior supporting edge and spread upward, tending to form arches. The function of the reinforcement is then to act as a tension rod, and under these conditions the required section seems to be very much smaller than the one calculated by prior methods. Special reinforcement should be provided at the supports, taking into account the local concentration of stress. The usual criterion seems then to be uneconomical. The proposed criteria have been applied in recently-constructed industrial buildings in Argentina,²⁴ and a satisfactory behavior has been observed, with a considerable saving of steel.

WILLIAM A. CONWELL,²⁸ M. ASCE.—The authors are to be commended for bringing to the publications of the ASCE the results of problems of great interest to the structural engineer. However, a real appreciation of what has been done requires reference to the work of two of the present authors and G. W. Morgan,⁸ in which they outline their methods. The writer believes that only with this work as a background can one assess the value of this paper. It would have been most fortuitous had both works been published under one cover.

²⁵ "Några Undersökningar Rörande Skivor Och Höga Balkar Av Armerad Betong," by H. Nylander and H. Holst, *Transactions, Royal Technical Univ., Stockholm, Sweden*, No. 2, 1946.

²⁶ "Vigas de gran altura: Algunos criterios para su cálculo y disposición de la armadura," by C. A. Sciammarella, *Hormigón Elástico*, Buenos Aires, Argentina, December, 1951.

²⁷ "Några Undersökningar Rörande Skivor Och Höga Balkar Av Armerad Betong," by H. Nylander and H. Holst, *Transactions Royal Technical Univ., Stockholm, Sweden*, No. 2, 1946, pp. 51-52.

²⁸ Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

²⁹ "Analysis of Deep Beams," by H. D. Conway, L. Chow, and G. W. Morgan, *Journal of Applied Mechanics*, ASME, Vol. 18, No. 2, June, 1951, pp. 163-172.

There are two distinct goals in this type of work. They are: (1) To provide methods with which the engineer can obtain, quickly, approximate solutions which are an aid to his judgment and better than rule-of-thumb procedures; and (2) to give the engineer data, based on rigorous research, upon which he can base his designs. The paper, with its predecessor,⁸ accomplishes the first end. Although it may not fully attain the second, particularly in the field of shear stresses (which can be important), it certainly indicates the direction in which considerably more work is needed.

One of the striking results of the paper is the close agreement between the authors' diagrams for shear and bending stresses for $\frac{H}{L} = \frac{1}{2}$, and those stresses obtained by the usual theory of flexure (see Figs. 2, 3, 4, and 5). It should serve to indicate that subsequent work might be directed toward problems having values of $\frac{H}{L}$ greater than $\frac{1}{2}$.

The authors indicate that the " * * * shear stress curves correspond to forces that are somewhat smaller than the actual shear force at the section" (see under the heading, "Results and Discussion"). By reading values from the shear curves for $\frac{H}{L} = 2$, the writer found errors in total shear of 8.4%, 11.8%, 15.2%, and 9.2% for Figs. 2(b), 3(b), 4(b), and 5(b), respectively. These percentages of error in total shear indicate that there are probably still greater errors in the shear stress at any point. Although the errors may seem large, the data remain valuable as an aid to judgment; and, although the authors charge the errors to the " * * * inaccuracy of the finite difference method" (see under the heading, "Results and Discussion"), there is no reason why the errors cannot be reduced substantially by using the finite difference method with a finer net. It may be necessary to introduce the finer net only in a part of the beam by methods developed by George H. Shortley, Royal Weller, and Bernard Fried.²⁹ The finer net would seem to be worthwhile in the region of sharp curvature of the shear stress curve, for example, in the region from $-0.3 H$ to $-0.5 H$.

A valuable addition to the paper (perhaps it could be included in the closing discussion) would be a contour plot of the principal stresses for at least one of the examples. Engineers are familiar with such plots, based on the usual theory of flexure and made in elementary strength of materials courses. A comparison with a contour plot obtained by the authors' method would be interesting.

It might be questioned whether, in the use of an expression as involved as Eq. 1, the authors have not lost some of the advantages usually attributed to the physical significance of the finite difference method. It is believed that considerable simplification would result in taking $\lambda = 1$, and treating with a special equation any rectangular net elements that might appear at the bound-

²⁹ "Numerical Solutions of Laplace's and Poisson's Equations," by George H. Shortley, Royal Weller, and Bernard Fried, *Bulletin No. 107*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio, September, 1940.

aries.²⁹ Still further, Eq. 1 is the difference equation equivalent to a fourth-order differential equation. The physical concept of such an equation often staggers the mind of an engineer who finds second-order equations (including partial derivatives) well within his grasp. Furthermore, the equation does not readily lend itself to the use of the numerical methods which can be very powerful in these problems.

The writer suggests the inclusion of a function,

$$u = \frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} \dots \dots \dots (13)$$

The resulting difference equations would be

$$u_{x,y} = \frac{u_{x-\Delta,y} + u_{x+\Delta,y} + u_{x,y-\Delta} + u_{x,y+\Delta}}{4} \dots \dots \dots (14)$$

$$Z_{x,y} = \frac{Z_{x-\Delta,y} + Z_{x+\Delta,y} + Z_{x,y-\Delta} + Z_{x,y+\Delta} + u_{x,y} \Delta^2}{4} \dots \dots \dots (15)$$

in which $h = k = \Delta$. The writer cannot say that it would be easier to solve simultaneously twelve equations of the type of Eqs. 14 and 15, rather than six of the more involved type of Eq. 1. However, it is apparent that Eqs. 14 and 15 lend themselves more readily to numerical computations.

As for physical significance, because $\nabla^2 Z = u$ and $\nabla^2 u = 0$, the sum of the angle changes in the x -direction and y -direction on the Z -surface equals u , and the sum of the angle changes in the x -direction and y -direction on the u -surface equals zero.

As an indication of the relative simplicity of the procedure which involves writing difference equations at two levels, the writer has selected from the previous paper by Messrs. Conway, Chow, and Morgan⁸ the array of Z -values for the problem treated there, which would go into Eq. 1.

				6.038
		5.440	4.754	2.778
3.500	4.000	3.500	2.000	-0.500
	2.560	2.246	1.222	
			0.962	

This array is composed of Z -values in terms of $\frac{P a^2}{32}$.

The corresponding sets of values for the two-level procedure at the same point are

Values of Z				Values of $u \Delta^2$		
			4.754			1.260
4.000	3.500	2.000		1.000	1.000	1.000
	2.246					0.740

All values in these two arrays are expressed in terms of $\frac{P a^2}{32}$. In each array, the value at the point being considered is the average of the four adjacent points—adjusted, in the case of Z , by the value $\frac{u \Delta^2}{4}$.

The writer considers it a privilege to discuss a paper into which the authors have put so much effort. He hopes that they will be able to continue the work they have begun so ably so that the profession will have the full benefit of their special talents.

HARRY D. CONWAY²⁰ AND GEORGE WINTER,²¹ M. ASCE.—The writers wish to thank Professors Guzmán and Luisoni and Mr. Conwell for their discussions. These contributions add materially to the worth of the paper.

The references to theoretical and experimental researches made in Argentina and Sweden are particularly interesting because they indicate the considerable practical importance of the subject. In so far as experimental research is concerned, the difficulty in obtaining a close approximation to uniform loading is considerable. As mentioned in the paper, theory shows that singularities exist in the shearing stresses at points on the boundary where the loading is discontinuous. In practice, the stress gradients are quite likely to be large at these points but they are hardly likely to be infinite. These and other practical considerations must be borne in mind when comparing theory with experiment, and may account for some apparent discrepancies.

The use of the Galerkin variational method by Messrs. Guzmán and Luisoni^{20,21} indicates an increase in accuracy with a decrease in labor. However, it is a little surprising to find that, in a later paper,^{22,23} they have reverted to the finite difference method. While on this subject, the writers would like to draw attention to a further method presented in the paper by two of the present authors and G. W. Morgan.⁸ An extension of this last method to beams of orthotropic material has been presented in a recent paper by one of the writers.³²

Mr. Conwell's summary, as well as his suggestions for increasing the accuracy of the shear stresses and possibly simplifying the numerical procedures, is valuable. These factors will be carefully considered when making further calculations.

The writers are not familiar with the Swedish²⁵ and Argentine²⁶ publications quoted by Messrs. Guzmán and Luisoni in spite of their extensive search of foreign literature. In addition, the second of these²⁶ was published subsequent to the writing of the present paper. The quotation from the paper by Messrs. Nylander and Holst reinforces the writers' contention that reinforcement should be arranged to counteract cracking, which cannot be achieved by concentrating tension reinforcement at the edge. If the latter is done, cracking converts the structure into an arch-like member whose action is entirely different from that predicted by deep-beam analysis. This, too, seems to be confirmed in the summary by Messrs. Guzmán and Luisoni of the Argentine²⁶ test results. Even though such arch action subsequent to extensive cracking is stated to require less reinforcement than computed by previous methods, the control over crack formation, possibly even at design loads, seems to be lost

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²² "Some Problems in Orthotropic Plane Stress," by Harry D. Conway, Paper No. 52-A-4, *Journal of Applied Mechanics*, ASME (publication pending).

in the process—a circumstance which is considerably more serious in deep beams than in shallow beams.

Also, because Messrs. Guzmán and Luisoni state that in these tests “*** failure occurs when the member has lost its resistance to compression in the supports ***,” it seems evident that, in view of this extraneous type of failure, full deep-beam (or arch) stresses leading to failure could not have been developed and, therefore, conclusions regarding them must be taken with reservations. The concentration of compression at, and near, the supports may require special measures in the form of reinforcement or even thickening of the section. Only when such local crushing has been eliminated will it be possible, in tests, to determine the effects of various types of reinforcement. Until such more extensive test evidence is available, the indication by Messrs. Guzmán and Luisoni of the importance of compression at the supports should be heeded in design.

In conclusion, attention may be drawn to another, very recent paper on this problem, which contains information that in part supplements that presented by the writers.³³

³³ “The Theory of Girder Walls with Special Reference to Reinforced Concrete Design,” by H. L. Uhlmann, *The Structural Engineer*, Vol. XXX, No. 8, London, England, August, 1952, p. 172.